Diversified Thinking.

Greater than the sum of its parts: looking at historic and future returns

The vast majority of investment decisions involve analysis of risk and return. Whereas measurement of risk typically enjoys a great deal of attention, appropriate measures of return are often not discussed.

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In this edition of Diversified Thinking, we look at two common ways – 'arithmetic' and 'geometric' – of calculating investment returns. We explain why these measures can produce significantly different results, how the results can differ over different time periods, and why it is important to understand this difference when making long-term projections. We also connect this discussion to the important related notion of a 'diversification bonus'.

Understanding the differences between arithmetic and geometric returns is important: for example, we believe that the difference between the two could easily be as much as 1% per annum on a typical equity portfolio. When compounded over a long period, this can have a significant effect on the perceived long-term outlook for an investor and the achievability of their objectives.

Managing expectations

We start with an example. Suppose that you are told that an asset returns either -10% or +10% each year, with 50% probability of each outcome. What outcome would you expect on an investment of £1,000 over 10 years?

One approach is just to take the average of all the end possible amounts, allowing for their chances of happening. If you do this calculation, it turns out that you will get an answer of £1,000. This seems like a logical result, given that the average return in any one year is 0% so the expected outcome at the end of each year is also £1,000, for as many years as you care to project. Using this method gives us what is called the 'mean' outcome.

However, there is another way of looking at the question. There are 10 years, and in each year we are equally likely to gain 10% or lose 10%. So it is reasonable to expect an average scenario with a gain of 10% in 5 of the years and a loss of 10% in the other 5 years. You could therefore argue that we should expect to have £1,000 x 1.10^5 x 0.90^5 = £950 at the end of 10 years. The likelihood of ending up with more than £950 is equal to the likelihood if ending up with less than £950. This is what is called the 'median' outcome.

The median outcome of £950 is lower than the mean outcome of £1,000, which means that in this example you are actually more likely than not to lose money. In fact, there is only a 38% chance that you would make a gain over 10 years, despite the mean outcome being equal to the amount of money you started with.

Skew

To understand why there is a less than 50% chance of making a gain. In this example, it is helpful to see how the possible outcomes are distributed. The best possible outcome over 10 years is £1,000 x 1.10^{10} = £2,594 and the worst outcome is £1,000 x 0.90^{10} = £349. There are two possibly surprising results here. First, how wide the range of outcomes

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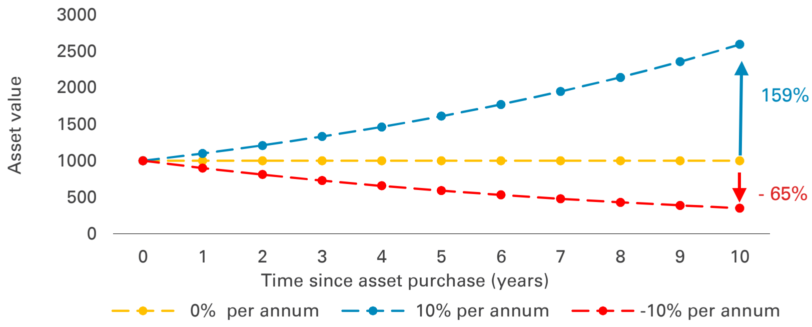
Figure 1. Exponential growth of assets



Source: LGIM

is, and second, the size of the possible gain (£1,594) relative to the size of the potential loss (£651). This is shown in Figure 1.

More formally, investment outcomes – particularly over long-term time periods – typically exhibit 'positive skew'. The typical pattern is shown in more detail in Figure 2 where we more realistically assume that the expected return is greater than 0% in any one year. Fewer than half the outcomes are better than the mean, but some of the gains are significantly bigger than any of the losses.

The very large positive outcomes have low probability, but exert a high impact on the mean – effectively pulling the mean up relative to the median. An important question is therefore: to what degree should investors allow their investment projections and decisions to be influenced by these unusual and extreme positive outcomes?

The answer depends on what the investor is trying to achieve. Holders of lottery tickets, for example, are only interested in rare extreme positive outcomes – the right tail of a very skewed distribution. However, long-term investors are not gamblers and are mostly interested in striking a balance, taking enough risk to earn a reasonable return over the long term under most scenarios, while avoiding (where possible) a high probability of dramatic loss.

Arguably, long-term investors should be less interested in the impact of the highly improbable but extreme outcomes, particularly positive ones, which can exert a significant force on the mean, and should instead be more interested in the central case, i.e. the median outcome. As the above example illustrates, a key issue with focusing too much on the mean outcome rather than the median is that over time, the actual return to the investor is increasingly likely to be less than the mean.

Arithmetic and geometric rates of return

What does this have to do with arithmetic and geometric rates of return? It turns out that the median is related to the geometric return, and the mean is related to the arithmetic one. We explain this further in the next few sections.

Geometric return

The geometric return is defined by looking at the constant annual rate of return that is implied by the start and end values of the portfolio. This method uses the exact start and end asset values, and the return is calculated as:

$$\text{Geometric Return} = \left(\frac{\text{End Asset Value}}{\text{Start Asset Value}}\right)^{1/\text{investment period (years)}} -1$$

For example, if you start with £100, invest for 10 years, and end with £200, your geometric return over the period is:

$$\left(\frac{200}{100}\right)^{1/10} -1 = 0.0718 = 7.18\% \text{ per annum}$$

For the calculation of the geometric return, it does not matter what the returns were in any particular year, or the pattern of returns across the different years. All that matters for the geometric return calculation is how much money you made over the whole period in question.

Arithmetic return

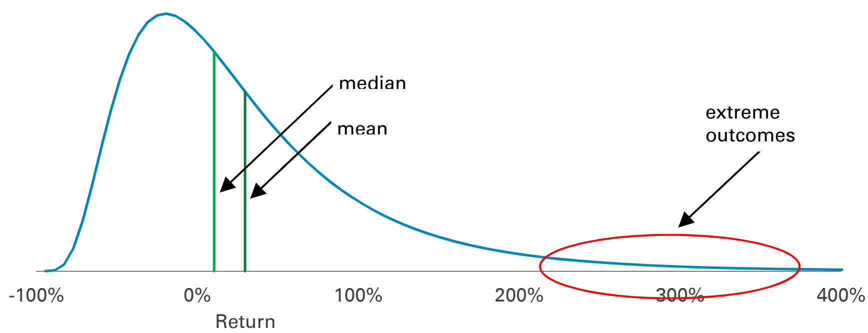
The arithmetic return is quite different. It can be defined as 'the sum of all the percentage returns over the individual years, divided by the number of years'. For example, if a) you receive a return of 10% in one year, and -10% in the next year, your arithmetic return over the two years is 0%. Similarly b) if you get a return of 0% in one year and 0% again in the next year, your arithmetic return is also 0%.

The arithmetic return, by contrast to the geometric, therefore does not have a direct relationship with the end asset value or how much money you made over the whole period. Both case a) and case b) have an arithmetic return of 0%, but in case a) you would end up with £1,000 x 1.1 x 0.9 = £990, whereas in case b) you would end up with £1,000. In case a), your geometric return works out as -0.50% per annum, and in case b) your geometric return works out as 0% per annum. This shows that the same arithmetic return for two different histories could imply different end asset values and hence two different geometric returns.

Historic returns

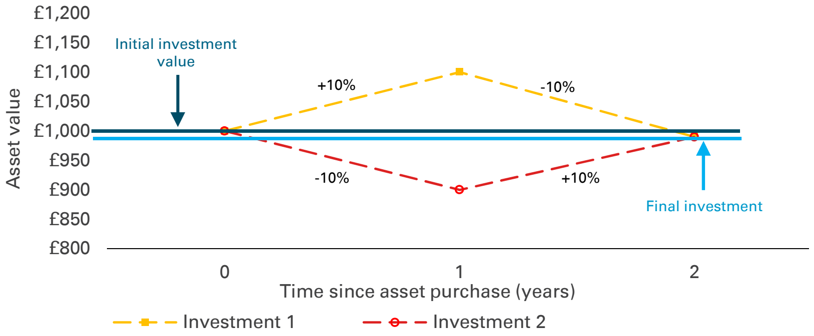
Investors are generally most interested in the return that represents the actual gain or loss that they made over the period of investment, i.e. the actual end asset value compared to the start asset value. The geometric return is therefore most appropriate and is by far the most widely used measure for historic returns.

Figure 2. Mean and median returns for a typical distribution of investment outcomes over a long period



Source: LGIM, illustrative purposes only

Figure 3. 10% gain/loss followed by a 10% loss/gain



Source: LGIM

Arithmetic returns and the mean outcome

As discussed earlier, the mean outcome at the end of a multi-year period is equal to the compounding of the expected arithmetic returns for each of the individual one-year periods[1]. An example of this was given at the start of this article; in this case, the mean one-year return for each year was 0%. Compounding ten 0% returns gives a 10-year compounded return of 0%. This tells us that the mean outcome is a 0% return over 10 years, hence a mean end portfolio value of £1,000.

However, the mean future outcome may not be the most useful measure of outcomes over longer-term time periods because of the skew, and specifically because of the influence on the mean of low-probability but extreme events. The median will usually be a more appropriate measure, but it will generally be lower than the mean.

A key consequence is that if you make accurate one year predictions of expected returns, year after year for many future years, and then compound these expected one-year returns, then you're likely to systematically overestimate the central case return over the long-term future.

By compounding in this way, you will be achieving an estimate that is closer to the mean than the median, and this mean will be skewed by some low probability but very positive events. If you are not careful, and are not aware of the dangers of combining individual year estimates into multi-year projections, you may end up being over-optimistic about the likely future long-term outcome from your investment strategy.

Assuming the returns varied from period to period, it can be shown that the geometric return of a historic period, for any asset, will always be lower than the arithmetic return. As a simple example, in Figure 3 we show two alternative sets of returns:

- firstly there is a return of +10% in the first year and -10% in the second
- or alternatively there is a return of -10% in the first year, followed by a return of +10% in the second year

In both cases the mean annual investment return is 0% per annum (the arithmetic return) but the constant rate of return is -0.50% per annum (the geometric return). The extent of the reduction in return as you move from an arithmetic to a geometric measure is called the 'volatility drag'. Essentially, in the example above, the drag arises because a loss of 10% and a gain of 10% (or vice versa) do not cancel each other out – you actually end up worse off.

The greater the volatility, the worse the volatility drag is – as shown in Figure 4. Had the returns in the example above been +20% and -20%, the final pot would have been only £960 rather than £990, and the geometric return would have been about -2.0% per annum, about four times worse, even though the arithmetic return would still have been 0%.

Virtually no-one focuses on the arithmetic return when looking back over history, but there is some merit to using expected arithmetic returns when it comes to looking to the future because this is related to the mean outcome. However, care is needed because the investor is less and less likely to achieve the mean outcome over time, as discussed earlier.

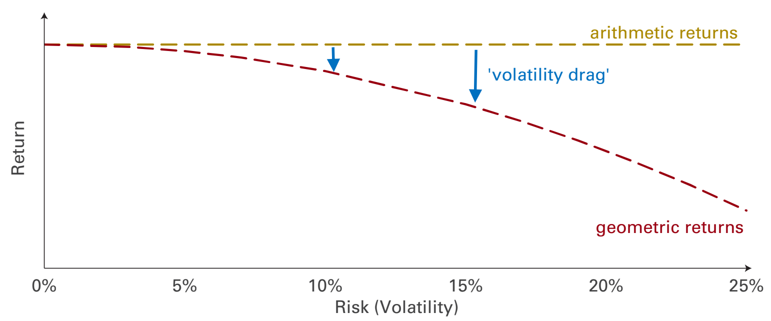
Expected future investment returns

Investors recognise that future returns are generally risky and uncertain. When they are making investment decisions (or funding decisions in the case of defined benefit pension schemes, or contribution decisions in the case of defined contribution pensions) people often start by making a best estimate of the future expected return, and then look at the risks associated with this.

For short periods, it doesn't matter much what the definition of best estimate or expected return is. Over a one-year period there is no difference between the arithmetic and geometric return, so distinguishing between these two measures of return is unnecessary.

However, over longer periods the picture is less clear, and the common lack of clarity when talking about expected returns can have far-reaching effects. It becomes much more important to be clear whether the future expected return being estimated / quoted is an expected arithmetic return, an expected geometric return, or indeed something else.

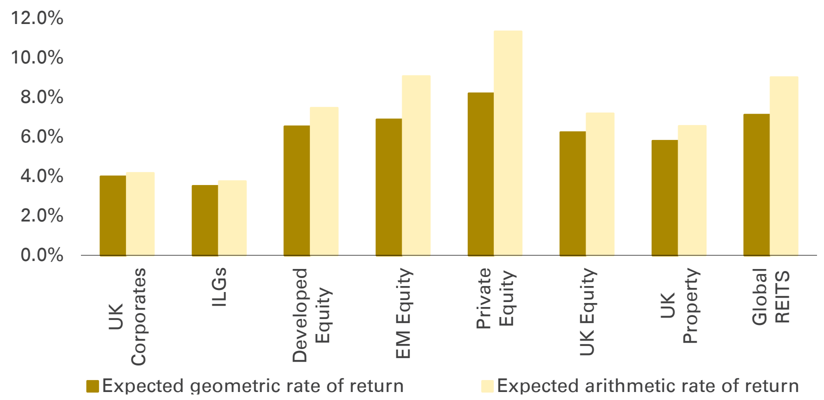
Figure 4. The volatility drag grows as the volatility increases



Source: LGIM, illustrative purposes only

[1]Strictly, this statement requires that the return distribution in each year is independent of the returns in each other year

Figure 5. Expected Geometric and Arithmetic rates of return over a 10-year projection



Source: LGIM

Geometric returns and the median outcome

It turns out that, over the long term, the expected geometric return corresponds closely to the median future outcome. This means that it closely reflects the central case, with equal chance of outperformance and underperformance.

Figure 5 shows expected arithmetic and geometric returns for some key asset classes, based on LGIM's long-term strategic asset allocation assumptions. As mentioned earlier, the difference, or volatility drag, reflects the level of volatility: it is higher for riskier assets (such as private equity, emerging markets equity) and lower for safer assets (such as bonds).

A worked example – developed market equities

Under LGIM's strategic expected return assumptions, for each individual year in the next decade our best estimate is that developed market equities will return approximately 4.6% above the risk-free rate in that year. However, this does not mean that over the full 10-year period our central case is that 4.6% per annum outperformance will occur. It is indeed the case that our mean-emphasis expected outcome for equities over 10 years is around 4.6% per annum above the risk-free return; however, over 10 years our central case, median-emphasis estimate, is a return of only 3.6% per annum above risk-free, because of the impact of volatility drag.

In Figure 5, we saw that volatility drag is higher for riskier underlying assets. It is also higher for longer time periods as illustrated in figure 6.

Another way of looking at this is to note that, over a 10-year period, to achieve a return of exactly 4.6% (that is, in line with expectations) in **each and every** one of the 10 years – which would lead to a return of 4.6% pa over the 10-year period – would in fact be a significantly better-than-average outcome overall, due to the unusual smoothness of this pattern of returns. A far more realistic pattern of returns, due to the volatile nature of equities, might be a return of say 19.2% for five of the 10 years, and -10.0% for the other five years. This still has the same average arithmetic return of 4.6% per annum, but it gives a 10-year annualised rate of return of only 3.6% per annum[2].

An important consequence of this argument is that returns that seem achievable over short-term time periods may be less achievable over long-term time periods.

This applies at a manager level as well as an asset class level. For example, a manager that has an objective of achieving outperformance of 4% per annum over a cash benchmark will need to target an expectation of more than 4% outperformance in any one year if they are to have a better-than-50% chance of achieving their long-term objective. This is because, unfortunately, they are likely to have years of underperformance as well as outperformance relative to their objective; the resulting volatility of their returns will drag down their median long-term outperformance. As a simple example, a manager that outperforms cash by 10% half the time and underperforms cash by 2% half the time will not outperform by 4% per annum overall, despite their expected outperformance being 4% in any single year.

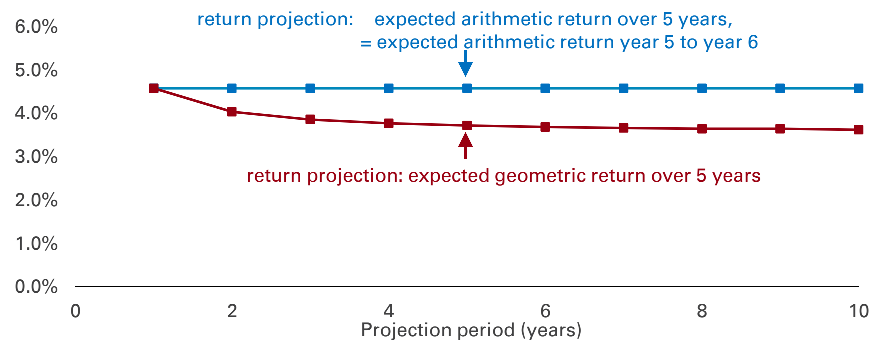
Combatting volatility drag – the diversification bonus

So, we have seen that volatility drags down median outcomes relative to the mean, reducing the central case return. Is there any way to mitigate this drag?

One simple solution is to use a diversified portfolio and regularly rebalance it. It is well known that portfolio theory suggests that diversification of assets can reduce the volatility of a portfolio. As we showed earlier, lower volatility will also have a beneficial impact on the expected geometric return. The reason for this is as follows:

- for a regularly rebalanced portfolio, the expected arithmetic return of the overall portfolio is the simple weighted average of the expected arithmetic returns of the underlying asset classes
- however, if volatility is lower due to diversification, the overall volatility drag of the portfolio

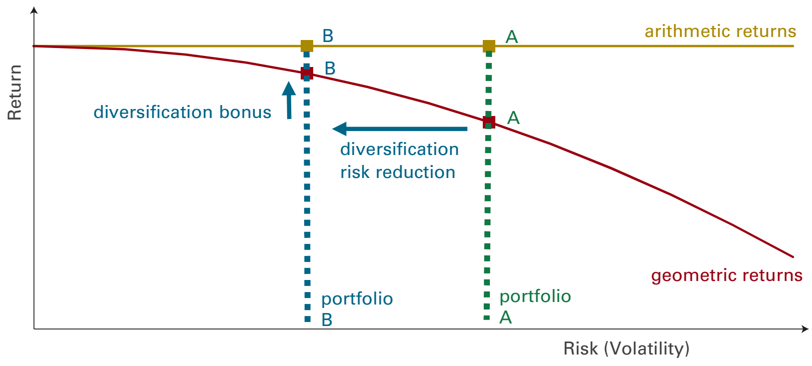
Figure 6. Developed market equities: Expected geometric and arithmetic rates of return (over the risk-free rate) over different projection periods



Source: LGIM

[2] In this paragraph, all returns are quoted relative to the risk-free rate. For simplicity, but the argument holds in broad terms even once the risk-free rate is added on top

Figure 7. The diversification bonus



Source: LGIM, illustrative purposes only

– the difference between the arithmetic and geometric returns of the portfolio – is smaller than you would expect if you simply weighted the volatility drags of the underlying constituents

This leads to a boost in the expected geometric return of the total portfolio, over and above the weighted average of the geometric returns of the underlying assets. This boost in the geometric return is known as the 'diversification bonus' and is shown in Figure 7. This builds on the Figure 4 and shows the arithmetic and geometric returns of two portfolios: A (less diversified) and B (with the same expected arithmetic return as A, but more diversified).

For both A and B, the volatility drag drops the arithmetic return (on the yellow line) down to the geometric return (on the red line). Because of its higher level of volatility, the volatility drag of portfolio A is larger than the volatility drag of B. This means that the reduction in expected geometric return is larger for A than B, even though they have the same arithmetic return. The reduction in risk through diversification gives a boost to geometric return of B, relative to the geometric return of A, which we call the diversification bonus.

As another more concrete example, consider a simple rebalancing strategy: 60% FTSE 100 Total Return Index and 40% Iboxx UK Corporate Index. In 1998-2013 the weighted average of the geometric rates of return for the two indices was 5.5%, whereas the regularly rebalanced strategy would have delivered a geometric return of 5.8%, resulting in the diversification bonus of 0.3%[3].

The inclusion of liabilities

The above discussion has focused on a purely asset-only world. However, many of the concepts we raise in this paper can, and most certainly should, be extended to include the liability angle: for instance, what is the impact of arithmetic versus geometric returns/discount rates in terms of the expected development of liability values over time? How do the expected arithmetic/geometric returns of the assets and liabilities interact? And by extension, what is the expected development of funding levels for defined benefit pension schemes under different investment and hedging strategies? Avoiding pitfalls in thinking in these areas is critical for choosing the right long-term investment strategy and maximising the chances of achieving long-term objectives. We will address these issues in a coming edition of Diversified Thinking.

Summary

- It is important to know what kind of return is considered – historic or expected, and what type of return has been quoted. A bold statement of the expected future return for an asset class, investment fund, or portfolio is meaningless unless it is clearly defined as an arithmetic return or a geometric one.
- The arithmetic rate of return can overstate the increase in wealth that has actually been achieved historically. Similarly, for prospective returns the arithmetic average can also give an unfair representation of the likely outcome due to the skewed nature of cumulative returns over the long-term. The expected geometric rate of return, whilst technically more challenging to use, is arguably a fairer measure, and a better estimate of the central case for longer-term investments.
- Diversification of a regularly rebalanced portfolio leads to a higher geometric rate of return than might be expected from combining the geometric rates of return of the portfolio's constituents. The additional geometric rate of return is called the diversification bonus.

[3]Index returns source: Bloomberg. Based on full calendar years

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